A Truncated RQ-iteration for Large Scale Eigenvalue Calculation

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Why RQ?

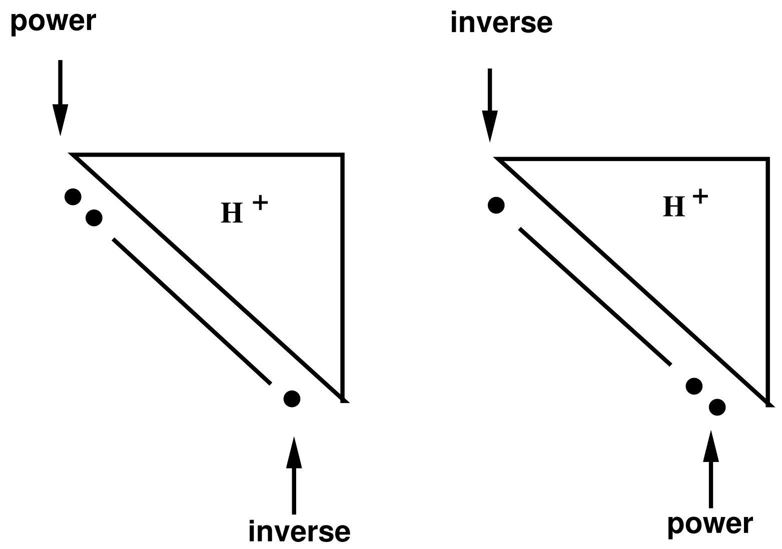
$$AV = VH$$

$$\begin{tikzcd}[column sep=small]
& AV = VH \arrow[dl] \arrow[dr] & \\
AV = VQR \arrow[d, Rightarrow] & & AV = VRQ \arrow[d, Rightarrow] \\
A(VQ) = (VQ)RQ \arrow[dr] & & AVQ^H = VQ^H(QR) \arrow[dl] \\
& AV^+ = V^+H^+ &
\end{tikzcd}$$

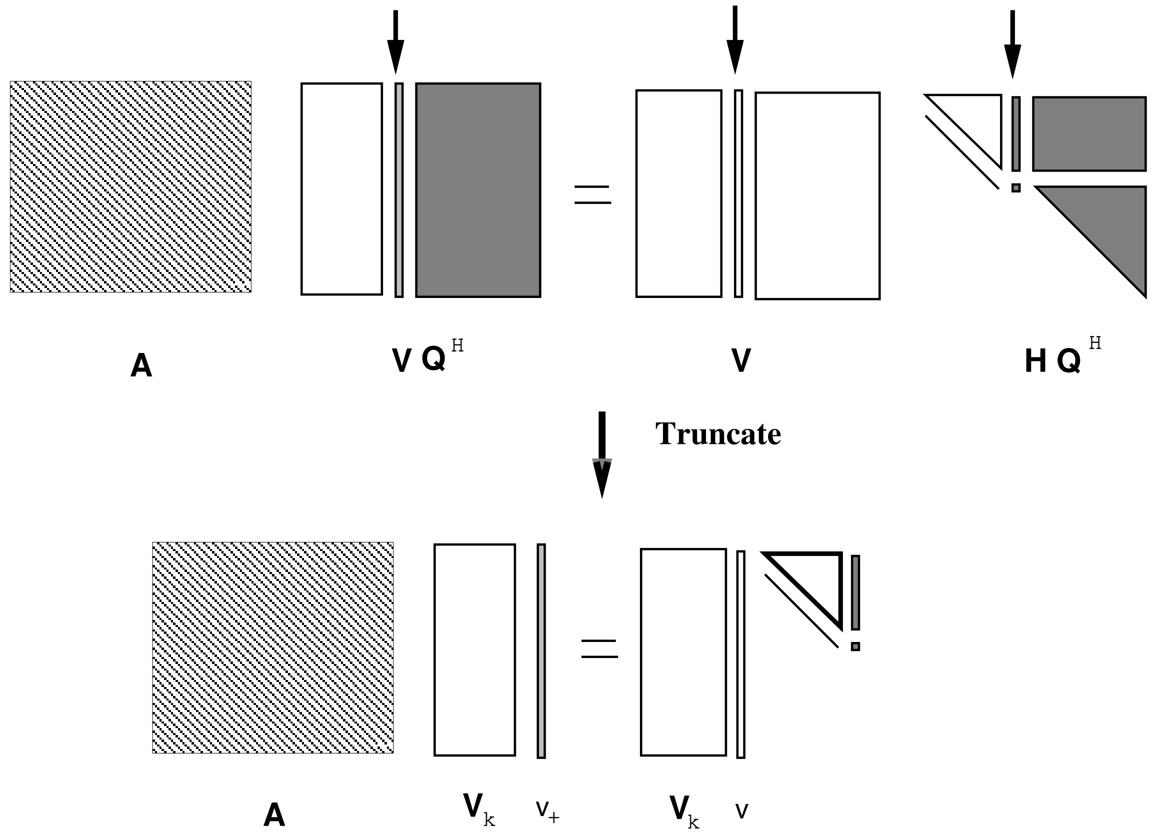
$$AV^+ = V^+H^+$$

Difference

QR	RQ
$AV = (VQ)R$	$A(VQ^H) = VR$
$\Downarrow$	$\Downarrow$
$Av_1 = v_1^+ \rho_{1,1}$	$Av_1^+ = v_1 \rho_{1,1}$



Truncating the RQ–iteration



Similar to the correction equations developed in Jacobi-Davidson, but V_k form an orthonormal basis of a Krylov subspace.

$$(I - V_k V_k^H)(A - \mu I)(I - V_k V_k^H)w = v\alpha$$

$$\begin{aligned} v_+ &\leftarrow \frac{w}{\|w\|} \\ h &\leftarrow V_k^H (A - \mu I) v_+ \\ \alpha &\leftarrow v^H (A - \mu I) v_+; \end{aligned}$$

Example

- The matrix A: 2-D discrete Laplacian (100×100);
- Target: smallest eigenvalues;
- $k = 4$;
- Convergence tolerance: 10^{-15};

$$\begin{matrix} \alpha_1 & \beta_1 & & & \\ \beta_1 & \alpha_2 & \beta_2 & & \\ & \beta_2 & \alpha_3 & \beta_3 & \\ & & \beta_3 & \alpha_4 & \beta_4 \\ & & & \beta_4 & \alpha_5 \end{matrix}$$